

Extragalactic Astronomy and Cosmology

Ten things to remember from the PHYS 316 course, fall 2006

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1 Hubble relation

Edwin Hubble found in the 1920s when studying galaxies and their redshifts ($z = \frac{\Delta\lambda}{\lambda_0} = \frac{1}{a} - 1$, where $\Delta\lambda$ is the difference in wavelength of a line in the spectrum compared to its laboratory wavelength λ_0 , and a is the scale factor) that there is a linear correlation between the receding speed of galaxies and their distance. This can be written in the form

$$v_r = H_0 \cdot d \quad (1)$$

where v_r is the radial velocity (km s^{-1}), d is the distance in Mpc and H_0 (in $H_0 = (73 \pm 3) \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the *Hubble constant*. This relation is only valid in the local Universe because the Hubble parameter is a function of time ($H(t)$). This observation is direct evidence for an expanding Universe.

2 Einstein's general relativity

In Newton's theory mass tells gravity how to exert a force and force tells mass how to accelerate ($F = ma$). Based on mind-experiments (like the light beam emitted in a gravitational field and in an accelerated lift) Albert Einstein used the equivalent principle to define the interaction of matter and space differently. In his theory, mass-energy ($E = mc^2$) tells space time how to curve, and curved space-time tells mass-energy how to move. Based on this ideas Einstein formulated the *Einstein field equations*. Their solutions are called *metrics*. The *Minkowski metric* is a solution to the Einstein field equation for flat space time: $ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2$. $ds^2 = 0$ represents the path of light, the *null geodesic*. The *Robertson-Walker metric* solves the Einstein field equation in curved space: $ds^2 = -c^2 dt^2 + a(t)^2 [dr^2 + S_\kappa(r)^2 d\Omega^2]$

3 Friedmann equation

The *Friedmann equation* is a special case of *Einstein's field equations* for the case of an expanding or contracting Universe. We learned several ways to write the Friedmann equation, for example:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} \epsilon - \frac{\kappa c^2}{R_0^2 a^2} + \frac{\Lambda}{3} \quad (2)$$

H is the *Hubble parameter*, a is the scale factor (for today we set $a(t_0) = 1$), ϵ is the energy density, κ the curvature (flat Universe: $\kappa = 0$, positively curved

$\kappa = +1$, negatively curved $\kappa = -1$), R_0 is the world radius, and Λ is the cosmological constant.

In order to take into account the different components we want to consider in the Universe, we can write the Friedmann equation in the following form:

$$\dot{a}^2 = \frac{8\pi G}{3c^2} \sum_{\omega} \epsilon_{\omega,0} a^{-1-3\omega} - \frac{\kappa c^2}{R_0^2} \quad (3)$$

Here ω being the equation of state parameter. This dimensionless parameter is $\omega = 0$ for non-relativistic matter, $\omega = \frac{1}{3}$ for radiation, and $\omega = -1$ for the cosmological constant.

4 Density parameter Ω

The density parameter Ω is defined as $\Omega = \frac{\epsilon}{\epsilon_c}$, where ϵ is the energy density and ϵ_c is the critical density. In the case that the energy density is equal to the critical density, we get $\Omega = 1$ and the Universe is flat. If the energy density is smaller than the critical one, we have a open Universe, with a larger energy density a closed Universe. Remember that the Universe does not change its κ parameter, i.e. once the Universe is flat, it will always be flat. We can write the Friedmann equation as a function of the density parameters of the single components:

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1 - \Omega_0}{a^2} \quad (4)$$

5 Three component Universe

We consider in our models three components for the Universe. Non-relativistic matter has no pressure ($P = \omega\epsilon = 0$), radiation has positive pressure, and dark energy has negative pressure. The three components have density parameters of $\Omega_m = 0.3$, $\Omega_{rad} = 5 \times 10^{-4}$, and $\Omega_{\Lambda} = 0.7$, respectively. This means that the Universe is flat and right now dominated by dark energy. Because of the negative pressure of the cosmological constant the Universe will continue expanding. When studying the importance of the different components throughout the history of the Universe, one has to keep in mind that $\epsilon_m \propto a^{-3}$, $\epsilon_{rad} \propto a^{-4}$, and the dark energy has a constant energy density. The scale factor is closely related to the redshift $a = \frac{1}{1+z}$ in *all* Universes described by the *Friedmann equation*. The early Universe was radiation dominated, then matter, and now, since $a \simeq 0.75$, dominated by dark energy. The matter in the Universe consists mainly of *dark matter* ($\Omega_{DM} = 0.26$). This matter has a mass, but no charge and is non-baryonic in nature. It can be detected through the rotation curves of galaxies and through the velocity dispersion of galaxies in clusters of galaxies. Baryonic matter can be found mainly in the gas which is trapped in galaxy clusters ($\Omega_{gas} = 0.04$), while the visible matter in stars and in the interstellar medium accounts for only $\Omega_* = 0.004$). The amount of baryonic matter we can detect is consistent with the value we get from the nucleosynthesis theory.

6 Measuring Distances

Measuring distances in the Universe is crucial for determining the true nature of our Universe. In order to measure the large distances, one first has to measure the local distances and then work the way up the *distance ladder*. Distances to nearby stars (up to 1 kpc) can be determined using the *parallaxic angle*. Some variable stars, the *Cepheids*, show a characteristic correlation between luminosity and period, thus can be used to determine their distance out to the nearest galaxies. Cosmological distances can be determined by using *Supernovae Type Ia* as *standard candles*. Supernovae Type Ia are caused by binary systems of a *white dwarf* and a “normal” star. A *white dwarf* is a star of solar type at the end of its life time. It is supported against gravitational collapse by *degeneracy pressure*. When this white dwarf is located in a binary system and accretes matter from the companion, it can reach the critical mass of $1.4M_{\odot}$ and collapse in a Supernova explosion. Because this always happens at the same mass, those explosions have always the same luminosity and thus can be used as standard candles.

7 Cosmic Microwave Background

The Cosmic Microwave Background (CMB) is a nearly isotropic background radiation which appears nowadays to have a temperature of $T_0 = 2.75\text{ K}$. The CMB radiation has a nearly perfect blackbody shape. It was created at the moment the Universe became transparent, some 350,000 years after the Big Bang at $z = 1100$ when the Universe had a temperature of $T_{ion} \simeq 3400\text{ K}$. This moment is called also the *last scattering*. At this moment photons decoupled from matter, because the free electrons got bound by protons to form hydrogen. Before this, photons interacted efficiently with electrons through *Thomson scattering* and the Universe was opaque. The CMB was extrapolating back to the moment when the CMB radiation was created, The COBE and WMAP satellite-experiments show that the CMB is nearly isotropic, with deviations of only $30\mu\text{K}$. The CMB shows anisotropies mainly on two different angles. At the 1 degree angle anisotropy is caused by density fluctuations of dark matter, at angles of about 0.6 degrees we see acoustic oscillations, caused by the baryon-photon fluid being affected by the density fluctuations. Both effects cause temperature fluctuations in the CMB, as measured by COBE and WMAP. The COBE results were so important for our understanding of the Universe, that Mather and Smoot were awarded the physics nobel prize for their work on COBE in 2006.

8 Nucleosynthesis

The basic elements of our Universe were created within the first 3 minutes after the Big Bang. The more heavy an elementary particle is, the earlier it was formed in the Universe. On the contrary, the more complex an atom nuclei is, the later in the Universe it was formed. Assuming equilibrium between protons

and neutrons in the beginning, the free neutrons started decaying into protons. Because this reaction is based on the nuclear *weak force*, it's cross-section is comparably low and thus it requires extreme high temperatures. After 1 sec the Universe had cooled down sufficiently so that neutron decay was not possible anymore, and the neutron-to-proton ratio was “frozen” to be $n_n/n_p \simeq 0.15$. In the universe then Deuterium (1 neutron and 1 proton in the core) was formed, followed by Tritium (2 neutrons, 1 proton) and Helium. Because the binding energy per nucleon in Helium is comparably high, this element was kind of a dead end in nucleo-synthesis. After about 3 minutes the temperature in the Universe was too low in order to perform a significant amount of fusion processes. Thus the element mix after 3 minutes stayed more or less constant until the first stars started to produce heavier elements (but this is after the CMB creation starting at times $t \gg 10^7$ yr). The Universe consisted mostly of protons (which will later catch electrons and form Hydrogen), ^4He , Deuterium (^2H), ^3He , and Tritium (^3H) (in decreasing abundance). Only traces of ^7Be and Lithium were created.

The fact, that apparently there is only matter and no antimatter in the Universe, points to a small overabundance of matter over antimatter in the very early Universe.

9 Inflation

The benchmark model alone, without inflation, has three major problems: the horizon problem (why is the temperature of the CMB the same even in places which had no contact throughout the life of the Universe), the flatness problem (why is $\Omega = 1$), and the question why there are no magnetic monopoles. Inflation solves these problems through an exponential inflationary period at the very beginning, which inflated sub-microscopic sizes to parsec scale. Through this, the Universe as we see it in the CMB, has been connected before inflation. Inflation also flattens the Universe. And it explains why there are no magnetic monopoles - the starting density of monopoles was lowered by inflation down to a number density which is too small to be detectable in the local Universe.

10 The Benchmark Model

The *benchmark model* describes our current understanding of the Universe. As stated above, we seem to live in a flat Universe which has been dominated by dark energy for about 4 Gyr and which will continue expanding forever (*Big Chill*). Important epochs in the Universe and properties at that time are given in Table 1. The density parameters for the components in the Universe today are listed in Table 2. One has to keep in mind that all these values are only true in case our benchmark model describes the Universe correctly.

Epoch	Redshift	T [°K]	Time [yr]	scale factor
neutron freeze-out	4×10^9	9×10^9	1 sec	
photodissociation		7.6×10^8	3 min	
radiation=matter	3570	9730	47,000	0.00028
recombination	1370	3740	240,000	0.00073
photon decoupling	1100	3000	350,000	0.0009
last scattering	1100	3000	350,000	0.0009
matter= Λ	0.33	3.7	9.8×10^9	0.75
Today	0	2.75	1.35×10^{10}	1

Table 1: Important epochs in the Universe (*Benchmark Model*.)

photons	$\Omega_{\gamma,0} = 5.0 \times 10^{-5}$
neutrinos	$\Omega_{\nu,0} = 3.4 \times 10^{-5}$
total radiation	$\Omega_{r,0} = 8.4 \times 10^{-5}$
baryonic	$\Omega_{bary,0} = 0.04$
dark matter	$\Omega_{dm,0} = 0.26$
total matter	$\Omega_{m,0} = 0.30$
dark energy	$\Omega_{\Lambda,0} \simeq 0.70$

Table 2: Density parameters in the local Universe (*Benchmark Model*)